EE 330 HW 12 Solutions

Problem 1

$$I_{D2} = \frac{W_2}{L_2}\frac{L_1}{W_1} I_{D1}$$

$$I_{OUT} = I_{D4} = \frac{W_4}{L_4}\frac{L_3}{W_3} I_{D3}$$

$$I_{D3} = -I_{D2}$$

$$I_{D1} = -250\mu A$$

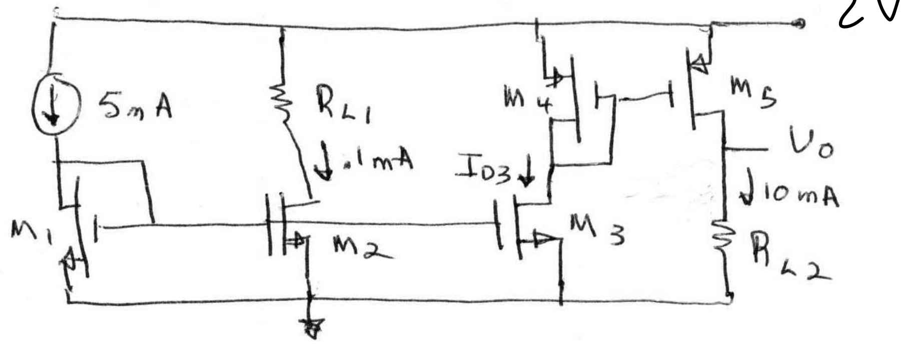
$$\Rightarrow I_{OUT} = \frac{W_4}{L_4}\frac{L_3}{W_3}\frac{W_2}{L_2}\frac{L_1}{W_1} \cdot 250\mu A$$

$$= \left(\frac{5}{1}\right)\left(\frac{6}{20}\right)\left(\frac{4}{2}\right)\left(\frac{1}{5}\right) \cdot 250\mu A$$

$$= 150\mu A$$

Problem 2

a) One solution



$$\left(\frac{W_2}{L_2}\right)\left(\frac{L_1}{W_1}\right)(5mA) = .1 mA$$

Let $W_1 = 50\mu$, $L_1 = 1\mu$

If $L_2 = 1\mu$, $W_2 = 1\mu$

Let $W_3 = W_1$
$L_3 = L_1$

$\therefore I_{D3} = 5mA$

$$\left(\frac{W_5}{L_5}\right)\left(\frac{L_4}{W_4}\right)(5mA) = 10mA$$

Let $W_4 = 40\mu$, $L_4 = 1\mu$

If $L_5 = 1\mu$, $W_5 = 80\mu$

b) To keep M5 in saturation

$$|V_{DS5}| > |V_{GS5} - V_{TH5}|$$

$$V_{DD} - V_O > |V_{EB5}|$$

but

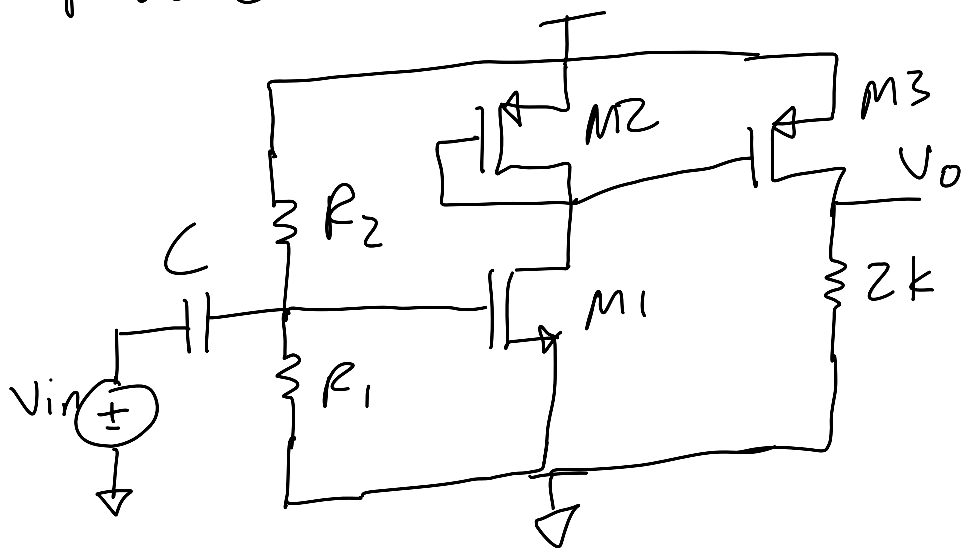
$$I_{D5} = \frac{\mu_p C_{OX} W_5}{2L_5}(V_{GS5} - V_{TH5})^2$$

$$|V_{GS} - V_{TH5}| = \sqrt{\frac{2 I_{DS}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$|V_{EB5}| = 1.73V$$

$$\therefore V_O < V_{DD} - |V_{EB5}| = 2 - 1.73 = \underline{270mV}$$

Problem 3



Assume C is very large

$$A_v = (-g_{m3})(2k) \times -\frac{g_{m1}}{g_{m2}}$$

$$A_v = (g_{m3})(2k)\left(\frac{g_{m1}}{g_{m2}}\right)$$

Lets set $(g_{m3})(2k) = 1$

$$\frac{g_{m1}}{g_{m2}} = 5$$

Consider the second stage:

$$g_{m3} = \frac{2 I_{D3}}{|V_{EB3}|} \Rightarrow \frac{2 I_{D3}}{|V_{EB3}|}(2k) = 1 \quad (V_{EB3} = V_{GS3} - V_{T3})$$

Let $|V_{EB3}| = 0.4V$

then, $I_{D3} = \frac{(0.4)(1)}{2(2k)} = 0.1mA = 100\mu A$

V_{OQ} will be $(0.1m)(2k) = 0.2V$

Using the square law model for saturation (ignoring λ effect)

$$\left(\frac{W}{L}\right)_3 = \frac{2 I_{D3}}{\mu_p C_{ox} |V_{EB3}|^2} = \frac{2(100\mu)}{\frac{250\mu}{3} \cdot 0.4^2} = 15$$

Let $L_3 = 1\mu m$, $W_3 = 15\mu m$

Consider the second stage

$|V_{EB2}| = |V_{EB3}|$ because M2/M3 have the same V_{GS} and same V_T

$$\frac{g_{m1}}{g_{m2}} = 5 \Rightarrow \mu_n C_{ox} \frac{W_1}{L_1} V_{EB1} = \left(\mu_p C_{ox} \frac{W_2}{L_2} V_{EB2}\right) 5$$

$$V_{EB1} = V_{EB2} \frac{\mu_p C_{ox}}{\mu_n C_{ox}} \left(\frac{\frac{W_2}{L_2}}{\frac{W_1}{L_1}} \right) 5$$

$$\frac{\mu_p C_{ox}}{\mu_n C_{ox}} = \frac{1}{3}$$

So,

$$\frac{\left(\frac{W_2}{L_2}\right)}{\left(\frac{W_1}{L_1}\right)} = \frac{V_{EB1}}{V_{EB2}} (3)\left(\frac{1}{5}\right)$$

Choose $V_{EB1} = 0.8\,V$

$$\frac{\left(\frac{W_2}{L_2}\right)}{\left(\frac{W_1}{L_1}\right)} = 1.2$$

Choose $L_2 = L_1 = 1\mu m$
$W_1 = 1\,\mu m$
$W_2 = 1.2\,\mu m$

Now, find R_1, R_2

$$V_{G1} = V_{EB1} + V_{TN} = 0.8 + 0.4 = 1.2\,V$$

$$\frac{R_1}{R_1 + R_2} 2 = 1.2$$

Let $R_1 = 100\,k\Omega$
Then $R_2 = 66.7\,k\Omega$

Problem 4 a)

$$\left.\begin{array}{l} I_{B1} + I_{B4} = \dfrac{2.5 - 0.6}{R_1} \\ I_{C1} = \beta I_{B1} \\ I_{C4} = \beta I_{B4} \end{array}\right\} \Rightarrow I_{C1} + I_{C4} = \beta \cdot \frac{1.9V}{R_1} \quad (1)$$

$$\left.\begin{array}{l} I_{C1} = J_S A_{E1} e^{\frac{V_{BE1}}{V_t}} \\ I_{C4} = J_S A_{E4} e^{\frac{V_{BE4}}{V_t}} \end{array}\right\} \Rightarrow$$

since $V_{BE1} = V_{BE4}$

$$I_{C1} = I_{C4} \frac{A_{E1}}{A_{E4}} \quad (2)$$

∴ from (1) and (2)

$$I_{C4} = \frac{\beta (1.9V)}{R_1 \left(1 + \frac{A_{E1}}{A_{E4}}\right)}$$

and

$$I_{C1} = \frac{\beta (1.9V)}{R_1 \left(1 + \frac{A_{E4}}{A_{E1}}\right)}$$

From the $Q_2 : Q_3$ current mirror

$$V_{O1} = I_{C1} \frac{A_{E3}}{A_{E2}} R_2 = \frac{\beta (1.9V)}{R_1 \left(1 + \frac{A_{E4}}{A_{E1}}\right)} \frac{A_{E3}}{A_{E2}} R_2$$

From the $M_5 : M_6$ current mirror

$$V_{O2} = (I_{C4}) \frac{W_6}{L_6} \frac{L_5}{W_5} R_3 = \frac{\beta (1.9V)}{R_1 \left(1 + \frac{A_{E1}}{A_{E4}}\right)} \frac{W_6}{L_6} \frac{W_5}{L_5} R_3$$

b)

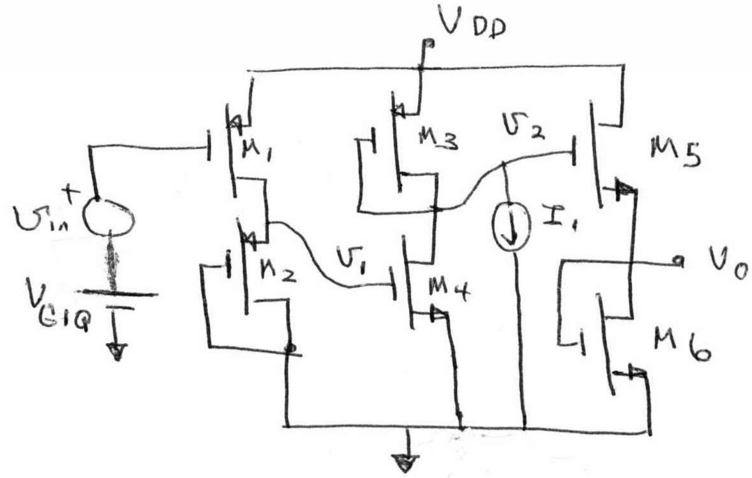
$R_2 = 10.1\ k\Omega$

$R_3 = 10.53\ \Omega$

(just solve equations above)

Problem 5

a)



$$\frac{v_o}{v_{in}} = \frac{v_o}{v_2} \cdot \frac{v_2}{v_1} \cdot \frac{v_1}{v_{in}}$$

$$\frac{v_o}{v_2} = -\frac{g_{m5}}{g_{m6}}$$

$$\frac{v_2}{v_1} = -\frac{g_{m4}}{g_{m3}}$$

$$\frac{v_1}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

$$\therefore \quad A_v = -\frac{g_{m1}}{g_{m2}} \cdot \frac{g_{m4}}{g_{m3}} \cdot \frac{g_{m5}}{g_{m6}}$$

b)

$$\frac{g_{m1}}{g_{m2}} = \frac{\frac{2 I_{DQ1}}{V_{EB1}}}{\frac{2 I_{DQ2}}{V_{EB2}}}$$ but $I_{DQ1} = I_{DQ2} \Rightarrow \frac{g_{m1}}{g_{m2}} = \frac{V_{EB2}}{V_{EB1}}$

but $V_{EB1} = 2 - 1 - |V_{TH1}| = \sout{.50}\ 0.6V$

$V_{EB2} = |V_{GS2}| - |V_{TH2}| = 1 - 0.4 = \sout{.50}\ 0.6V$

$$\therefore \frac{g_{m1}}{g_{m2}} = \frac{0.6}{0.6} = 1$$

$$\frac{g_{m4}}{g_{m3}} = \frac{\frac{2 I_{DQ4}}{V_{EB4}}}{\frac{2 I_{DQ3}}{V_{EB3}}}$$ but $I_{DQ3} = I_1 + I_{DQ4} = 2 I_{DQ4}$

so $\frac{g_{m4}}{g_{m3}} = \frac{V_{EB3}}{2 V_{EB4}}$ but $V_{EB3} = V_{DD} - V_{GS} - |V_{Tp}| = 2 - 1.3 - 0.4 = 0.3V$

$V_{EB4} = V_{GS4} - V_{Tn} = 1V - 0.4 = 0.6V$

$$\therefore \frac{g_{m4}}{g_{m3}} = \frac{0.3V}{(2)0.6V} = 0.25V$$

$$\frac{g_{m5}}{g_{m6}} = \frac{\frac{2 I_{DQ5}}{V_{EB5}}}{\frac{2 I_{DQ6}}{V_{EB6}}}$$ but $I_{DQ5} = I_{DQ6} \Rightarrow \frac{g_{m5}}{g_{m6}} = \frac{V_{EB6}}{V_{EB5}}$

but $V_{EB5} = V_{GS5} - V_{Tn} = 1.3 - .7 - 0.4 = 0.1V$

$V_{EB6} = V_{GS}$

but $V_{EB5} = V_{GS5} - V_{Tn} = (1.3 - .7) - ~~.5~~\ 0.4 = ~~0.3V~~\ 0.2V$

$V_{EB6} = V_{GS6} - V_{Tn} = .7V - ~~.5V~~\ 0.4 = ~~0.3V~~\ 0.3V$

$\therefore \quad \dfrac{g_{m5}}{g_{m6}} = \dfrac{0.3V}{0.2V} = 1.5$

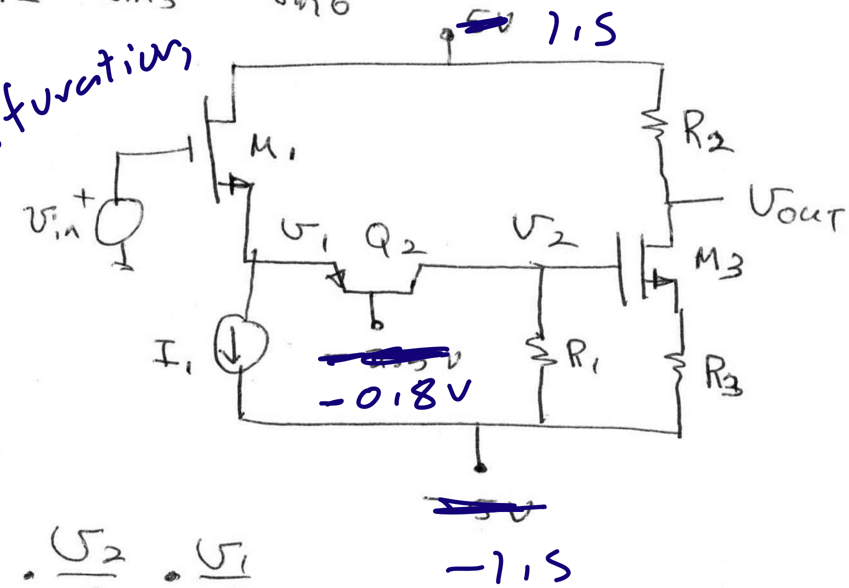
It thus follows that

$$A_v = -\frac{g_{m1}}{g_{m2}} \cdot \frac{g_{m4}}{g_{m3}} \cdot \frac{g_{m5}}{g_{m6}} = -1 \cdot (0.25) \cdot (1.5) = -0.375$$

Problem 6

Assume MOS are in saturation

BJT is in Forward Active



$$\frac{v_o}{v_{in}} = \frac{v_o}{v_2} \cdot \frac{v_2}{v_1} \cdot \frac{v_1}{v_{in}}$$

$$\frac{v_o}{v_2} = -\frac{R_2}{R_3} \qquad \frac{v_2}{v_1} = g_{m2} R_1 \qquad \frac{v_1}{v_{in}} \simeq 1$$

$$\therefore \quad \frac{v_o}{v_{in}} \simeq -\frac{R_2}{R_3} \cdot g_{m2} R_1$$

Problem 7

Assume Q_2 is in Forward Active, then

$V_{EB2} = 0.6V$

$V_{S1} = -0.8 + 0.6 = -0.2V$

$V_{GS1} = 0 - (-0.2) = 0.2V < V_{TN}$ so

M1 is in cutoff

If M1 is in cutoff, $V_{S1} = -1.5V$

(because of the current source)

The p-n junction E-B is now reverse biased so the assumption is invalid.

Assumption 2: Q_2 is in cutoff

now, V_{in} has no effect on V_{out}

$V_{G3} = 0$ because it is pulled low by R_1

Thus, $V_{GS} = 0 - 0 = 0$ and M3 is in cutoff

V_{out} is then pulled high by R_2

$V_{out} = 1.5V$, $\frac{V_{out}}{V_{in}} = 0$

This was an accident. Q_2 was supposed to be in forward active but we messed up the problem.

If $V_{B2} = -1.3\,V$ then,

$$V_{S1} = -1.3 + 0.6 = -0.5$$

$$V_{GS1} = 0 - (-0.5) = 0.5$$

$$I_{D1} = \frac{\mu_n C_{ox}}{2} \left(\frac{8}{2}\right)(0.5 - 0.4)^2 = 5\,\mu A$$

Then,

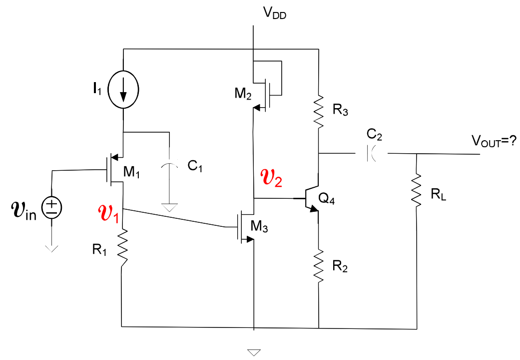
$$I_{CQ2} = 200\,\mu A - 5\,\mu A = 195\,\mu A$$

$$g_{m2} = \frac{195\,\mu A}{25\,mV} = 7.8\,mS$$

From #6,

$$\frac{V_o}{V_{in}} = -\frac{R_2}{R_3} g_{m2} R_1 = -\frac{20}{12.5}(7.8m)(8k) = \underline{\underline{-99.84}}$$

Problem 8



$$A_V = \frac{v_{OUT}}{v_{IN}} = \frac{v_{OUT}}{v_2} \bullet \frac{v_2}{v_1} \bullet \frac{v_1}{v_{IN}}$$

The last state is a CE with RE stage. The middle stage is a CS stage and the first stage is a CS stage. Thus from the last state

$$\frac{v_{OUT}}{v_2} = -\frac{R_3 // R_L}{R_2}$$

$$R_{IN3} \simeq \beta R_2$$

From the middle stage

$$\frac{v_2}{v_1} = -g_{m3} \bullet \left(\frac{1}{g_{m2}} // R_{IN3} \right)$$

And from the first stage

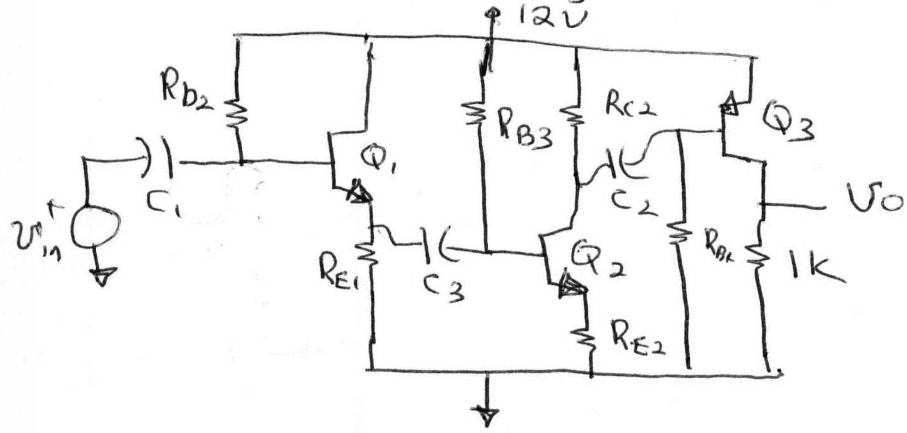
$$\frac{v_1}{v_{IN}} = -g_{m1} R_1$$

Thus the overall gain is

$$A_V = -\frac{R_3 // R_L}{R_2} - g_{m3} \bullet \left(\frac{1}{g_{m2}} // \beta R_2 \right) \bullet g_{m1} R_1$$

Problem 9 One solution:

3-stage structure. Stage 1: CC to get large input impedance
Stage 2: Inverting Gain Stage   Stage 3: Inverting gain stage



$A_v = 1 \cdot (-g_{m3})(1k) \cdot \left(-\frac{R_{C2} // R_{B4} // r_{\pi 3}}{R_{E2}}\right)$

$R_{in} = R_{B2} // \beta[R_{E1} // \beta R_{E2}]$

Assume C_1, C_2, C_3 large

- Set $g_{m3} \cdot 1k = 60 \Rightarrow \frac{I_{C3}}{V_t} \cdot 1k = 60 \Rightarrow I_{C3} = 1.5mA$

$\therefore I_{B3} = \frac{I_{C3}}{100} = 15\mu A \Rightarrow \frac{12 - .6}{R_{B4}} = 15\mu A \Rightarrow R_{B4} = 626K$

- Set $(R_{C2} // R_{B4} // r_{\pi 3}) / R_{E2} = 1$

Observe $R_{IN3} = R_{B4} // r_{\pi 3} = 626K // \frac{V_t}{I_{B3}} \simeq 1.66K$

Pick $R_{C2} = 2K \Rightarrow \frac{2K // 1.66K}{R_{E2}} = 1 \Rightarrow R_{E2} = 907\Omega$

Pick $I_{C2} = 2mA \Rightarrow V_{B2} = .6 + (2mA)(907\Omega) = 2.4V$

$\therefore \frac{12 - 2.4}{R_{B3}} = \frac{I_{C2}}{\beta} \Rightarrow R_{B3} = \frac{(9.6)\beta}{2mA} = 480K\Omega$

- Pick $R_{E1} = 5K$, $I_{E1} = 1mA$

$\therefore V_{B1} = R_{E1} \cdot I_{E1} + .6V = 5.6V$

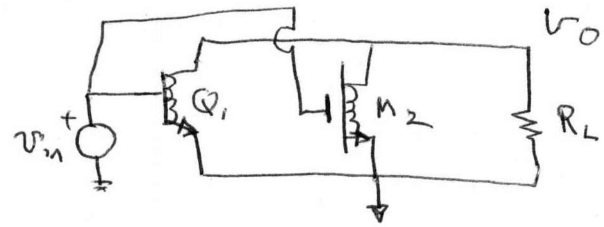
$I_{B1} = \frac{I_{E1}}{\beta} = \frac{12 - 5.6}{R_{B2}} \Rightarrow R_{B2} = 640K$

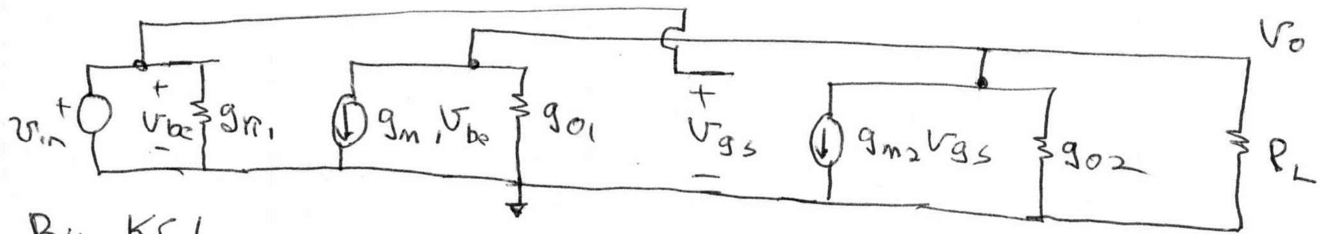
Check $R_{IN} = R_{B2} // \beta(R_{E1} // \beta R_{E2}) = 640K // (100 \cdot [5K // 90.7K])$

$R_{IN} = 272K\Omega$

Problem 10

a)





By KCL

$$v_o (g_L + g_{o1} + g_{o2}) + g_{m1} v_{in} + g_{m2} v_{in} = 0$$

$$\therefore \frac{v_o}{v_{in}} = - \frac{(g_{m1} + g_{m2})}{g_L + g_{o1} + g_{o2}} \simeq -(g_{m1} + g_{m2}) R_L$$

b) Observe $I_{CQ} = |I_{DQ}|$

$$I_{CQ} = \beta \frac{(0 - (-5 + 0.6))}{R_B} = \beta \cdot \frac{4.4V}{200k} = 2.2mA$$

$$g_{m1} = \frac{I_{CQ}}{V_t} = \frac{2.2mA}{25mV} = 8.8E-2$$

$$g_{m2} = \frac{2|I_{DQ}|}{|V_{EB2}|} = \frac{4.4mA}{1.8 - |V_{THP}|} = \frac{4.4mA}{1.05} = 4.2E-3$$

$$\therefore A_v = -(8.8E-2 + 4.2E-3)(1K) = -92.2$$

so $v_o(t) = (-92.2)(.01 \sin 1000t) = -.92 \sin(1000t)$